

The effects of public transport on urban form

Basso, Silva & Navarro

Introduction

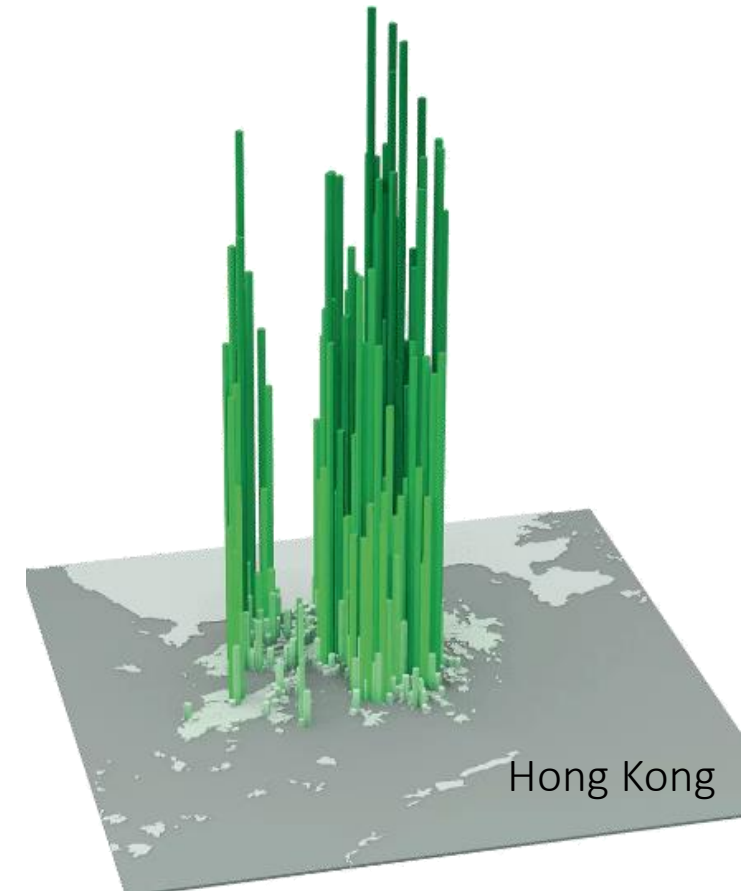
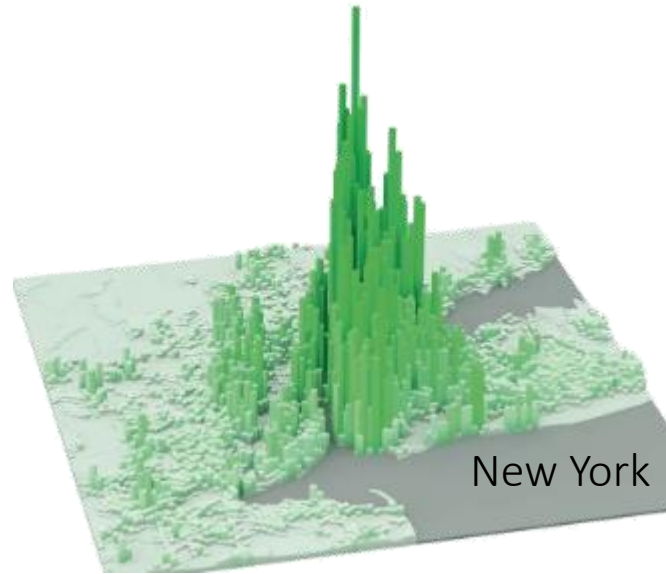
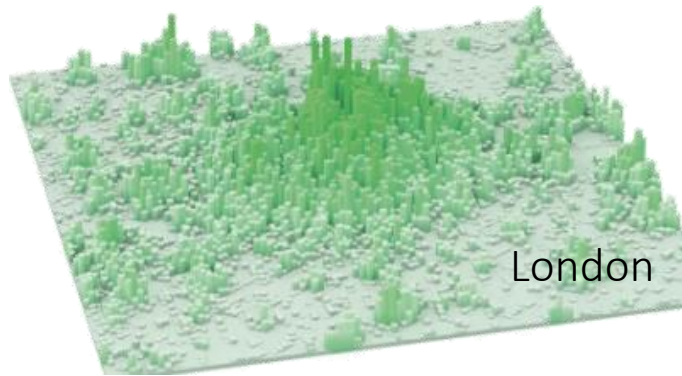
Motivation

Literature review

Hypothesis and Objectives

Motivation

Why do cities take different spatial forms?



Differences in height of residential buildings, rental price of housing, land rent, population density and urban boundary.

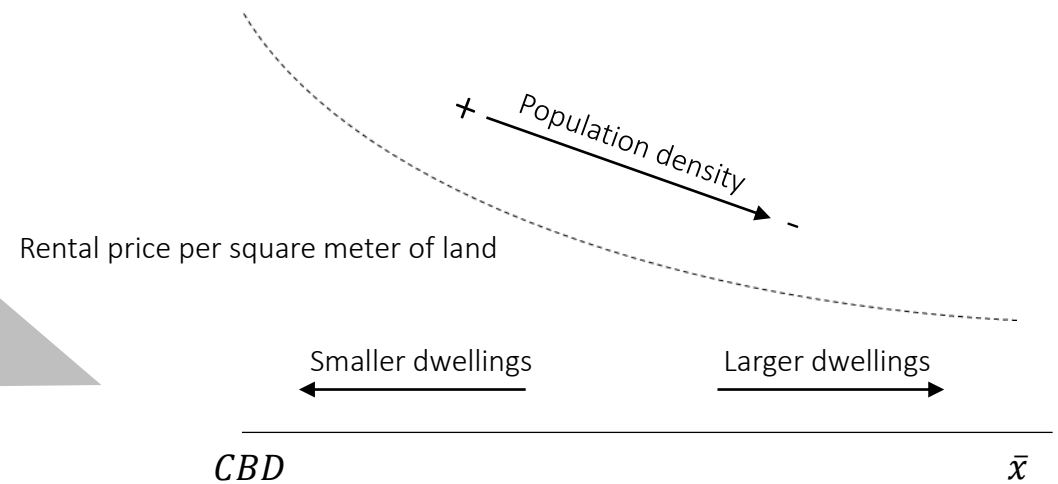
Which is the role of transport in these characteristics?

Literature review

Urban economics literature has answered this question with different approaches



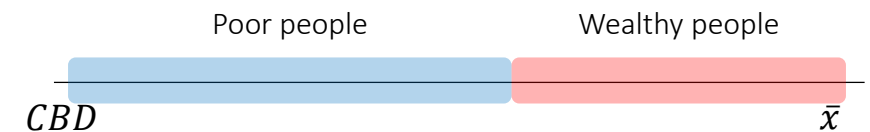
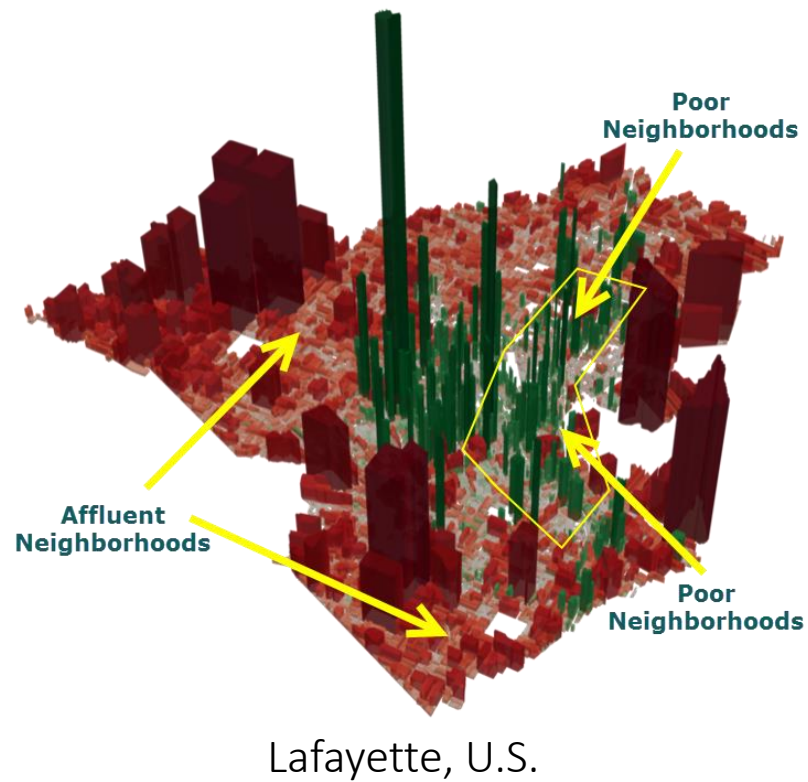
Atlanta, U.S.



Alonso (1964), Mills (1967) & Muth (1969)

Literature review

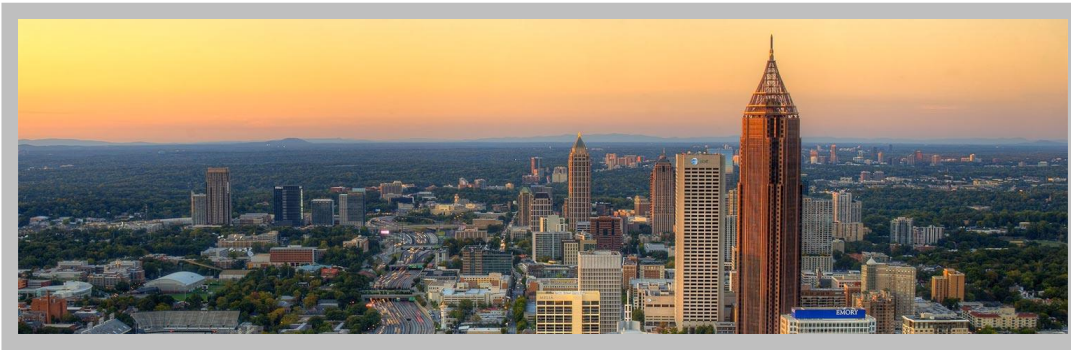
Urban economics literature has answered this question with different approaches



Alonso (1964), Wheaton (1976),
LeRoy & Sonstelie (1983)

Literature review

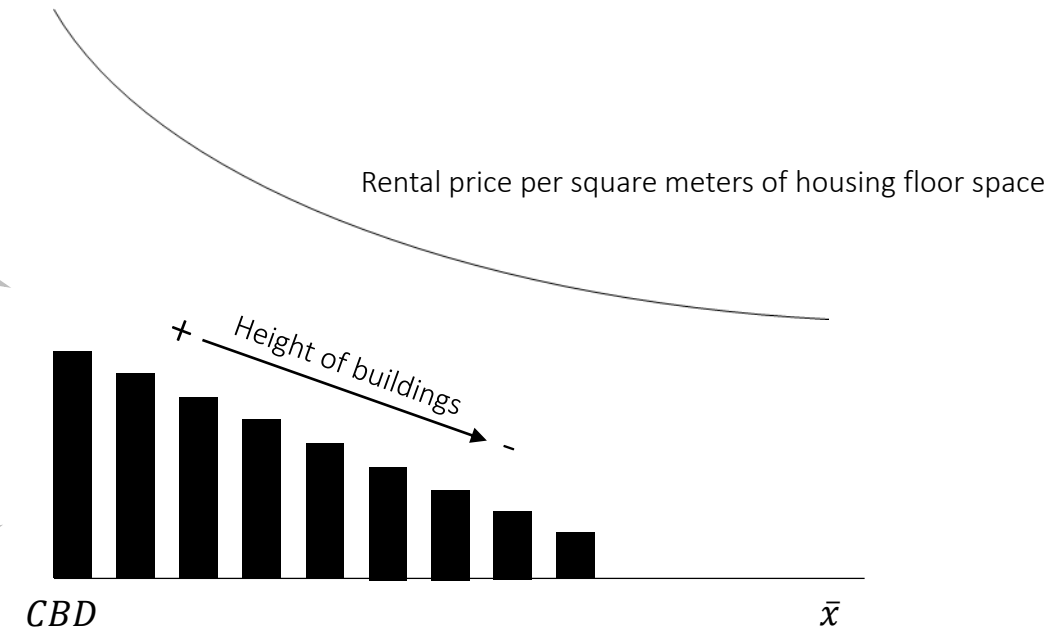
Urban economics literature has answered this question with different approaches



Atlanta, U.S.



Dallas, U.S.



Brueckner (1987)

Literature review

Limitations and remaining questions

Only private cars

What would the effect of [public transport](#) be in the urban form?

Only congestion pricing

Are there other [policies](#) to use when public transport is included?

Most papers consider homogeneous consumers

Which is the behavior of [heterogeneous consumers](#) in presence of public transport?

Hypothesis and Objectives

Including **public transport** and its interaction with other markets may add **new insights** to current literature conclusions.

Intermediate step

Spatial equilibrium model considering **only public transport** (subway) and homogeneous consumers.

Final step

Spatial equilibrium model considering **two modes of transportation**, policies and heterogeneous consumers.

Model

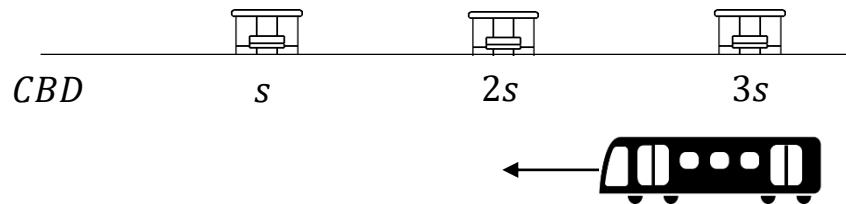
Description

First-order conditions

Stations

Description of the model

Consumer side



Transport network is radial and stations are equally spaced.

The highest utility level attainable at each location equals u

Monocentric city (all jobs are in the CBD) and homogeneous consumers (preferences, income rate w , and number of working hours per day j).

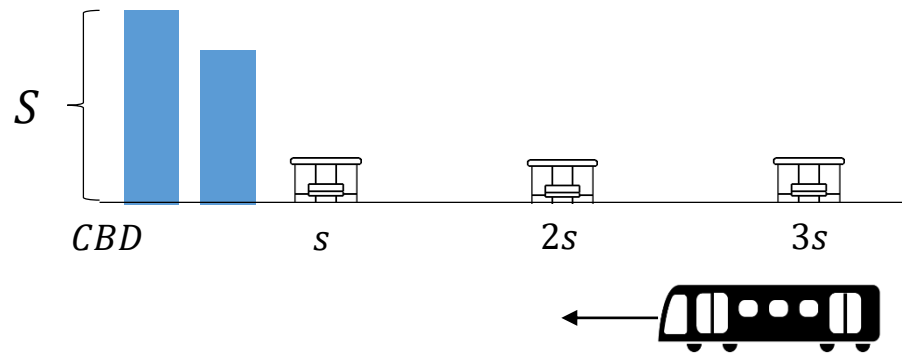
Utility function: $U(c; q; l)$ where c is consumption of a composite non-housing good, q is consumption of housing and l is leisure time.

Budget constraint: $wj = c + pq + e$ where p is the rental price per m² of housing and e is the fare of public transport.

Time constraint: $T = j + l + t$ where t is the total commuting time from x to the CBD.

Description of the model

Supply side



Durability is avoided.

S is the capital-land ratio N/l and denotes the height of buildings.

Housing is produced with inputs of land L and capital N , according to the concave constant returns function $H(N, L)$ which gives the number of m² of floor contained in a building.

The revenue from a building is $pH(N, L)$.

The costs are rL , where r is the land rent per m² and iN , where i is the rental price per unit of capital.

First-order conditions

Consumer side

$$\max U(c; q; l) = u$$

$$\text{s.t. } c + pq = y - \rho \text{ \& } T = j + l + t$$

$$\Leftrightarrow \max V(w(T - l - t) - pq - e; q; l) = u$$

(1) Consumer equilibrium

$$\frac{V_q(w(T-l-t)-pq-e;q;l)}{V_c(w(T-l-t)-pq-e;q;l)} = p$$

$$\frac{V_l(w(T-l-t)-pq-e;q;l)}{V_c(w(T-l-t)-pq-e;q;l)} = w$$

(2) Spatial equilibrium

$$V(w(T - l - t) - pq - e; q; l) = u$$

Supply side

$$\pi = pH(N, L) - iN - rL$$

$$= L(pH\left(\frac{N}{L}, 1\right) - i\frac{N}{L} - r)$$

$$= l(ph(S) - iS - r)$$

(1) Max profit for fixed l , choosing S

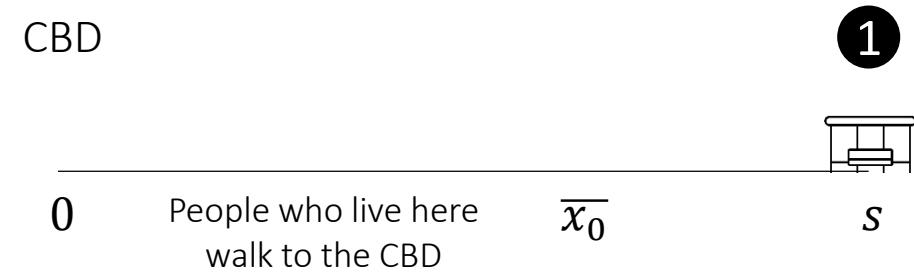
$$ph'(S) = i$$

(2) Zero-profit

$$ph(S) - iS = r$$

Stations

Between first station and CBD



$$\rho(s) = \gamma \frac{\bar{x}_0}{v_c}$$

$$e_s + \theta \int_0^s t_f ds + \gamma \frac{s - \bar{x}_0}{v_c} = \gamma \frac{\bar{x}_0}{v_c}$$

Fare from
station at s

Time in
vehicle from
station at s

Walking time
to station at s

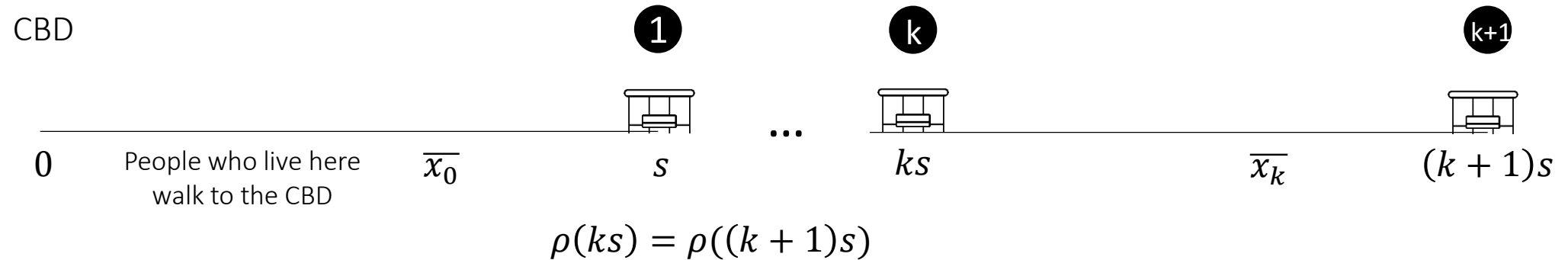
Walking time
to the CBD

It implies that $\bar{x}_0$ will
always be at the
right of the middle
of the section.

$$\bar{x}_0 = \frac{v_c}{2\gamma} (e_s + \theta t_f s) + \frac{s}{2}$$

Stations

Between two stations



$$e_{ks} + \theta \left[\int_0^{ks} t_f ds + t_b \int_{\bar{x}_0}^{\bar{x}_{k-1}} x \phi D dx \right] + \gamma \frac{\bar{x}_k - ks}{v_c} = e_{(k+1)s} + \theta \left[\int_0^{(k+1)s} t_f ds + t_b \int_{\bar{x}_0}^{\bar{x}_k} x \phi D dx \right] + \gamma \frac{(k+1)s - \bar{x}_k}{v_c}$$

$$\bar{x}_k = \frac{v_c}{2\gamma} \cdot \left([e_{(k+1)s} - e_{ks}] + \theta \left[t_f s + t_b \int_{\bar{x}_{k-1}}^{\bar{x}_k} x \phi D dx \right] \right) + \frac{s}{2} (2k+1)$$

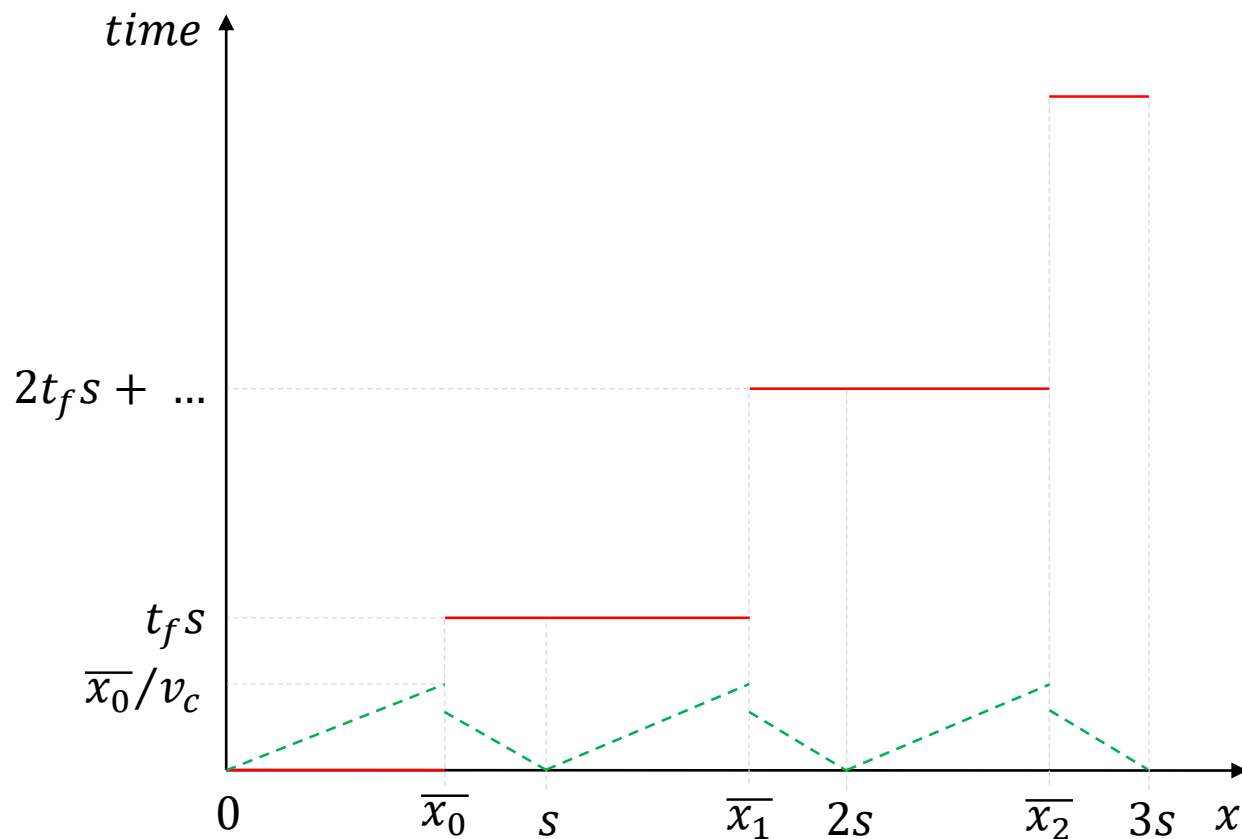
$\bar{x}_1$ can be obtained recursively using $\bar{x}_0$.

Difference between fares Time in vehicle between stations Boarding time in station k To locate x between stations k and $k+1$

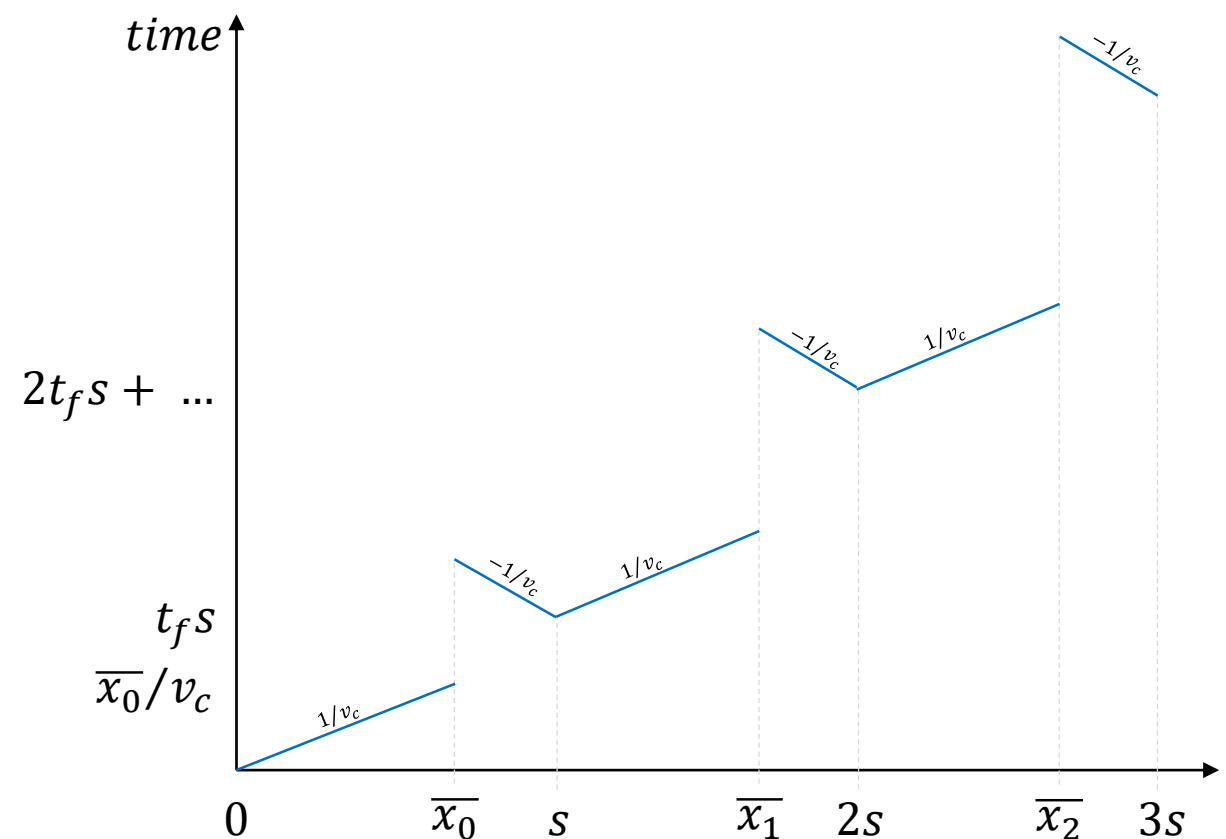
Stations

Effect on the commuting time

Time in vehicle and Walking time



Commuting time



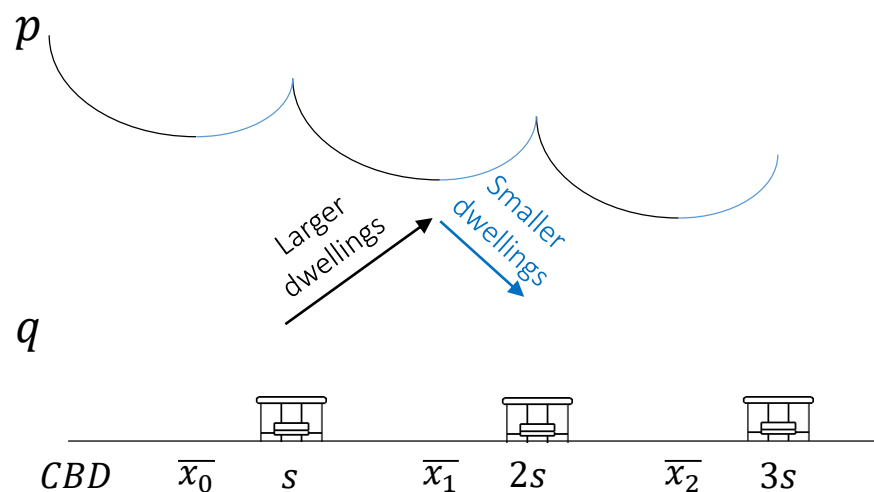
Description of equilibrium

Rental price of housing (p) and Consumption of housing (q)

Land rent (r) and Structural density (S)

Description of equilibrium

Rental price of housing (p) and Consumption of housing (q)



From the first-order conditions we can obtain:

$$\textcircled{1} \quad \frac{\partial p}{\partial x} = -\frac{1}{q} \left(w \frac{\partial t}{\partial x} + \frac{\partial e}{\partial x} \right) = \begin{cases} ks < x < \bar{x}_s \rightarrow \frac{\partial t}{\partial x} > 0 \rightarrow \frac{\partial p}{\partial x} < 0 \\ \bar{x}_s < x < (k+1)s \rightarrow \frac{\partial t}{\partial x} < 0 \rightarrow \frac{\partial p}{\partial x} > 0 \end{cases}$$

$$\textcircled{2} \quad \frac{\partial q}{\partial x} = \left[\frac{\partial \left(\frac{V_2}{V_1} \right)}{\partial q} - \frac{\frac{\partial \left(\frac{V_2}{V_1} \right)}{\partial l} \frac{\partial \left(\frac{V_3}{V_1} \right)}{\partial q}}{\frac{\partial \left(\frac{V_3}{V_1} \right)}{\partial l}} \right]^{-1} \frac{\partial p}{\partial x} = \eta \frac{\partial p}{\partial x} = \begin{cases} ks < x < \bar{x}_s \rightarrow \frac{\partial p}{\partial x} < 0 \rightarrow \frac{\partial q}{\partial x} > 0 \\ \bar{x}_s < x < (k+1)s \rightarrow \frac{\partial p}{\partial x} > 0 \rightarrow \frac{\partial q}{\partial x} < 0 \end{cases}$$

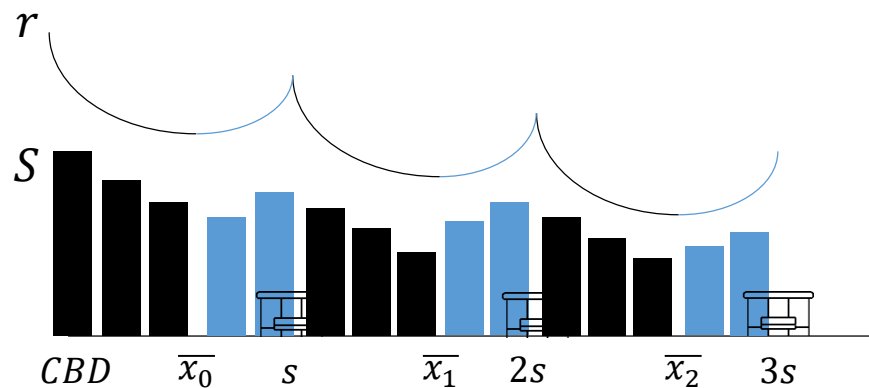
Description of equilibrium

Land rent (r) and Structural density (S)

Also, from the first-order conditions we can obtain:

$$\textcircled{3} \quad \frac{\partial r}{\partial x} = h(S) \frac{\partial p}{\partial x} = \begin{cases} ks < x < \bar{x}_s \rightarrow \frac{\partial p}{\partial x} < 0 \rightarrow \frac{\partial r}{\partial x} < 0 \\ \bar{x}_s < x < (k+1)s \rightarrow \frac{\partial p}{\partial x} > 0 \rightarrow \frac{\partial r}{\partial x} > 0 \end{cases}$$

$$\textcircled{4} \quad \frac{\partial S}{\partial x} = -\frac{\frac{\partial p}{\partial x} h'(S)}{ph''(S)} = \begin{cases} ks < x < \bar{x}_s \rightarrow \frac{\partial p}{\partial x} < 0 \rightarrow \frac{\partial S}{\partial x} < 0 \\ \bar{x}_s < x < (k+1)s \rightarrow \frac{\partial p}{\partial x} > 0 \rightarrow \frac{\partial S}{\partial x} > 0 \end{cases}$$



Conclusions

Summary

Ongoing research

Summary

These preliminary results show that public transport gives [additional insights](#) to explain the structure of cities.

Specifically, the [walking time](#) to public transport stations generates [discontinuous effects](#) in commuting times and urban form.

[The research is in progress](#). Understanding better the equilibrium, doing a comparative static analysis and testing our results empirically is necessary.

Ongoing research

Develop a comparative static analysis adding equilibrium conditions related to the [city boundary](#) and the [population](#).

Ex: How does the size of the city change due to the change of:

Fares: Plain, Depending on distance, Depending on zones.

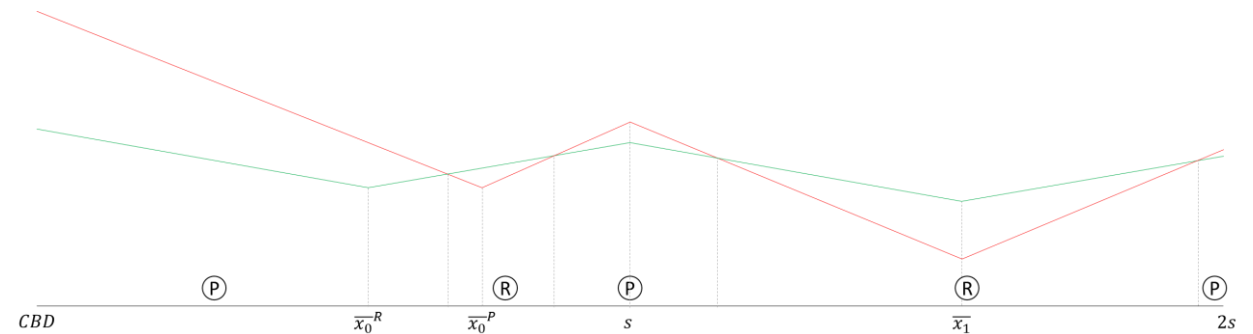
Wage rates: Bigger or smaller income inequalities.

Commuting times: Faster or slower transport modes.

Ongoing research

Include **different income groups** and find the equilibrium location pattern of poor and rich households along the city.

Our guess: In presence of public transport, high and low income groups may now be mixed along the city.



Ongoing research

Include both, private cars and public transport, jointly and evaluate the effect of [transportation policies](#) on urban form.

Ex: How does the size of the city change due to the implementation of:

Bus lanes.

Road pricing.

Transit subsidies.

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